Discrete Mathematics Recitation Course 1

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About Myself

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1-1

Propositional Logic

1-1 Ex.2

- Which of these are propositions? What are the truth values of those that are propositions?
  - a) Do not pass go. x
  - b) What time is it? x
  - c) There are seven days in a week. true
  - d) $4 + x = 5$. x
  - e) The moon is made of green cheese. false
  - f) $2^n \geq 100$. x

1-1 Ex.10

- Let *p*, *q*, and *r* be the propositions
  - *P*: You get an A on the final exam.
  - *q*: You do every exercise in this book.
  - *r*: You get A in this class.
  - Write the propositions using *p*, *q*, and *r* and logical connectives
- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class. $p \wedge \neg q \wedge r$
- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class. $(p \wedge q) \rightarrow r$
- f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

$r \leftrightarrow (p \vee q)$

1-1 Ex.28 - f)

- Construct a truth table for the compound proposition.

$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

1-1 Ex.52

- Are the system specifications consistent?
  - If the file system is not locked, then new messages will be queued. $\neg L \rightarrow Q$
  - If the file system is not locked, then the system is functioning normally, and conversely. $\neg L \leftrightarrow N$
  - If new messages are not queued, then they will be sent to the message buffer. $\neg Q \rightarrow B$
  - If the file system is not locked, then new messages will be sent to the message buffer. $\neg L \rightarrow B$
  - New messages will not be sent to the message buffer. $\neg B$
- Yes! Sol: first we better have *B* false…

1-2

Propositional Equivalences

1-2 Ex.16

- Show that $p \leftrightarrow q$ and $(p \land q) \lor (\neg p \land \neg q)$ are equivalent.

- Both propositions are true only when p and q have the same truth value

1-2 Ex.30

- Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

- This will be false only when $T \rightarrow F$
- Suppose that $(q \vee r)$ is false
- Then $(p \vee q) \wedge (\neg p \vee r)$ is false
- So we know the proposition is a tautology
- Or you can construct a truth table to prove

1-2 Ex.61

- Explain how an algorithm for determining whether a compound proposition is satisfiable can be used to determine whether a compound proposition is a tautology.

- To determine whether c is a tautology apply an algorithm for satisfiability to $\neg c$
- If $\neg c$ is satisfiable, then c is not a tautology, and conversely.

SAT Application

- For an AND2 gate with inputs a, b and output c, its CNF formula can be obtained by:

$$(a \wedge b) \leftrightarrow c$$

$$= ((a \wedge b) \rightarrow c)(c \rightarrow (a \wedge b))$$

$$= (\neg a \vee \neg b \vee c)(\neg c \vee (a \wedge b))$$

$$= (\neg a \vee \neg b \vee c)(\neg c \vee a)(\neg c \vee b)$$

1-3

Predicates and Quantifiers

1-3 Ex.2

- Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?
  - a) $P(orange)$ true
  - b) $P(lemon)$ false
  - c) $P(true)$ false
  - d) $P(false)$ true

1-3 Ex.16

- Determine the truth value of each of these statements if the domain for all variables consists of all real numbers.
  - a) $\exists x(x^2 = 2)$ true
  - b) $\exists x(x^2 = -1)$ false
  - c) $\forall x(x^2 + 2 \geq 1)$ true
  - d) $\forall x(x^2 \neq x)$ false

1-3 Ex.30

- Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y where x is 1, 2, or3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.
  - a) $\exists x P(x, 3)$
  - c) $\exists y \neg P(2, y)$

- $P(1,3) \vee P(2,3) \vee P(3,3)$
- $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

1-3 Ex.52

- As mentioned in the text, the notation $\exists! xP(x)$ denotes "There exists a unique x such that $P(x)$ is true." If the domain consists of all integers, what are the truth values of these statements?
- a) $\exists! x(x > 1)$ false
- b) $\exists! x(x^2 = 1)$ false
- c) $\exists! x(x + 3 = 2x)$ true
- d) $\exists! x(x = x + 1)$ false

1-3 Ex.62

- Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.
  - a) No ducks are willing to waltz. $\forall x(P(x) \rightarrow \neg S(x))$
  - b) No officers ever decline to waltz. $\forall x(R(x) \rightarrow S(x))$
  - c) All my poultry are ducks. $\forall x(Q(x) \rightarrow P(x))$
  - d) My poultry are not officers. $\forall x(Q(x) \rightarrow \neg R(x))$

1-4

Nested Quantifiers

1-4 Ex.28

- Determine the truth value of each of these statements if the domain of each variables consists of all real numbers.
- h) $\exists x \exists y (x + 2y = 2 \land 2x + 4y = 5)$ false
- i) $\forall x \exists y (x + y = 2 \land 2x - y = 1)$ false (only one sol.)
- j) $\forall x \forall y \exists z (z = (x + y) / 2)$ true

1-4 Ex.32 - b)

- Express the negation of the statement so that all negation symbols immediately precede predicates.

$$\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$$

- $\neg\big(\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)\big)$
- $\equiv \neg\exists x \exists y P(x, y) \vee \neg\forall x \forall y Q(x, y)$
- $\equiv \forall x \neg\exists y P(x, y) \vee \exists x \neg\forall y Q(x, y)$
- $\equiv \forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$

1-5

Rules of Inference

1-5 Ex.6

- Use rules of inference to show that the hypotheses
- "If it does not rain or it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," $(\neg r \vee \neg f) \rightarrow (s \wedge l)$
- "If the sailing race is held, then the trophy will be awarded," $s \rightarrow t$
- and "The trophy was not awarded" $\neg t$
- imply the conclusion "It rained." r

Step	Reason
1. $\neg t$	Hypothesis
2. $s \rightarrow t$	Hypothesis
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Hypothesis
5. $(\neg (s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition, using (3)
8. $r \wedge f$	Modus ponens using (6) and (7)
9. r	Simplification using (8)

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p, $p \rightarrow q$, $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\neg q$, $p \rightarrow q$, $\therefore \neg p$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$p \rightarrow q$, $q \rightarrow r$, $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$, $\neg p$, $\therefore q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism
p, $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$, $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification

1-5 Ex.20

- Determine whether these are valid arguments.
  - a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number. invalid
  - b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

valid; it is modus ponens

1-6

Introduction to Proofs

Proof Methods Review

- **Prove *p→q***
  - *Direct Proof*
    - 1. The first step: Assume *p is true*
    - 2. … rules of inference …
    - 3. The final step: *q must also be true*
  - *Proof by Contraposition*
    - Prove ***-q → -p***
  - *Proof by Contradiction*
    - Lec1 P.69

Proof Strategies

- Generally, if the statement is a conditional statement,
  - first try *direct proof*,
  - then try *proof by contraposition*,
  - finally try *proof by contradiction*

1-6 Ex.5

- Prove that if $m+n$ and $n+p$ are even integers, where m, n, and p are integers, then $m+p$ is even. What kind of proof did you use?

- *Direct proof*
  - Suppose $m+n=2s$ and $n+p=2t$
  - $m+2n+p=2s+2t$
  - $m+p=2(s+t-n)$

1-6 Ex.8

- Prove that if *n* is a perfect square, then *n*+2 is not a perfect square

- ***Direct proof***
  - Suppose that $n=m^2$
  - if m=0, than $n+2=2$
  - else $(m+1)^2 = m^2+2m+1 = n+2m+1 > n+2\cdot1+1 > n+2$
- Less obvious!

1-6 Ex.14

- Prove that if x is rational and $x \neq 0$, then $1/x$ is rational

- <u>*Direct proof*</u>
  - $x = p/q$, where p, $q \neq 0$
  - $1/x = q/p$ → rational

1-6 Ex.16

- Prove that if *m* and *n* are integers and *mn* is even, then *m* is even or *n* is even

- <u>*Proof by contraposition*</u>
  - if *m* and *n* are both odd
  - then *mn* is odd (This can be proved)

1-6 Ex.18

- Prove that if n is an integer and $3n+2$ is even, then n is even using
  - a) a proof by contraposition
  - b) a proof by contradiction
- a) If n is odd, write $n=2k+1$, then $3n+2 = 3(2k+1)+2 = 6k+5$, which is odd
- b) Suppose $3n+2$ is even and n is odd,

  If n is odd, then write $n=2k+1$, then $3n+2 = = 6k+5$, which is odd, a contradiction